DEVELOPING CRITICAL THINKING THROUGH THE USE OF REAL-LIFE APPLICATIONS

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<table>
<thead>
<tr>
<th>Title</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beginning Algebra</td>
<td>Connecting Concepts through Applications</td>
</tr>
<tr>
<td>Intermediate Algebra</td>
<td>Connecting Concepts through Applications</td>
</tr>
<tr>
<td>Beginning and Intermediate Algebra</td>
<td>Connecting Concepts through Applications</td>
</tr>
</tbody>
</table>
INTRODUCTIONS

- Please use chat window to introduce yourself. In particular, state your school and state.
POLL QUESTIONS

Question: How many webinars have you participated in?

a) None, this is my first.

b) 1 or 2 times.

c) 3 or more times.
WHAT IS CRITICAL THINKING?

• “Critical thinking is the intellectually disciplined process of actively and skillfully conceptualizing, applying, analyzing, synthesizing, and/or evaluating information gathered from, or generated by, observation, experience, reflection, reasoning, or communication, as a guide to belief and action” (Scriven, 1996).

• "Critical thinking is the ability to think about one's thinking in such a way as

1. To recognize its strengths and weaknesses and, as a result

2. To recast the thinking in improved form" (Center for Critical Thinking, 1996c).
How do we know when our students are thinking critically in mathematics courses?
WHAT IS CRITICAL THINKING? (CONTINUED)

• Students should be able to
  • Determine what it is they are being asked to do
  • Analyze the problem type
  • Apply the appropriate techniques
  • Determine if their answer makes sense
WHY TEACH CRITICAL THINKING?

• The Common Core State Standards sets the following goals (among others) for K-12 mathematics students

1. Make sense of problems and persevere in solving them

2. Reason abstractly and quantitatively

3. Construct viable arguments and critique the reasoning of others
WHY TEACH CRITICAL THINKING?  
(CONTINUED)

• Student Learning Outcomes (SLO’s) were driven at the federal level by the 2006 Spellings Commission (DOE) for a greater demand of accountability and transparency in higher education.

• State governments and accreditation agencies also are pushing for accountability and transparency through the use of SLO’s.
At Palomar Community College, one of the institutional student learning outcomes is “critical and creative thinking.”
WHY TEACH CRITICAL THINKING? (CONTINUED)

- The rubric for Palomar’s SLO on critical thinking is looking for the following skills:
  - Identify and understand the problem and issues
  - Analyze
  - Strategize
  - Draw conclusions and predict related outcomes
WHY TEACH CRITICAL THINKING? (CONTINUED)

### Palomar College General Education Outcomes
GE/Institutional SLO: Critical and Creative Thinking

**Definition:** Critical thinking is the intellectually disciplined process of actively and skillfully conceptualizing, applying, analyzing, synthesizing, and/or evaluating information gathered from, or generated by, observation, experiences, reflection, reasoning, or communication, as a guide to belief and action. The Foundation for Critical Thinking

**NOTE** about using this rubric: Evaluators are encouraged to assign a zero to any work sample that does not meet the emerging level.

<table>
<thead>
<tr>
<th></th>
<th>1 Emerging</th>
<th>2 Developing</th>
<th>3 Proficient</th>
<th>4 Exemplary</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Identify and understand the problem and issues</strong></td>
<td>• Cannot identify and/or describe the problem or issue</td>
<td>• Demonstrates some minimal or simplistic understanding of the problem</td>
<td>• Identifies, demonstrates, and/or describes the problem clearly with effective support</td>
<td>• Demonstrates, states, and/or describes the issue or problem clearly, delivering all relevant information necessary for full understanding</td>
</tr>
<tr>
<td><strong>Analyze</strong></td>
<td>• Presents a completely limited and/or biased perspective</td>
<td>• Presents a limited, biased perspective</td>
<td>• Analyzes and presents a point of view with a comparative perspective that includes other points of view without bias</td>
<td>• Analyzes, discusses and/or demonstrates comparative perspectives with full understanding of multiple positions</td>
</tr>
<tr>
<td><strong>Strategize</strong></td>
<td>• Presents no plan or solution, or uses illogical solutions to a problem</td>
<td>• Identifies possible solutions without clarity or creative thinking</td>
<td>• Identifies and develops alternative solutions</td>
<td>• Formulates a creative, original, and well-stated solution using sophisticated thinking</td>
</tr>
<tr>
<td><strong>Draw conclusions &amp; predict related outcomes</strong></td>
<td>• Does not tie conclusions to the information presented</td>
<td>• Does not consistently tie conclusions to the information presented</td>
<td>• Ties conclusion to a range of information including opposing viewpoints</td>
<td>• Uses logical conclusions and related outcomes to reflect a well-informed evaluation</td>
</tr>
</tbody>
</table>

Palomar College Learning Outcomes Council
• Given that the broad consensus is that critical thinking is a desirable trait to develop in our students, what are we doing to promote critical thinking in our mathematics courses?
HOW WE CAN TEACH CRITICAL THINKING

What we can do:

• Make sure students understand what the problem is asking them to do
• Make sure students focus on what type of problem they are being asked to solve or simplify
• Use applications that force students to think carefully about what their answers mean in a real-world context
• Have students look for the “reasonableness” of their answers
• Have students look for errors in other student’s work
• Reinforce throughout the course important concepts that students need to take from our classes (such as units)
• Have students write complete sentence answers to application problems
WHAT DO WE WANT OUR STUDENTS TO TAKE FROM THEIR MATH CLASSES?

• What do we want our students to take from their math classes?

• How can we find out?
WHAT DO WE WANT OUR STUDENTS TO TAKE FROM THEIR MATH CLASS?

- We want our students to have basic numeracy, the ability to measure correctly, to convert units, to perform algebraic operations, to round accurately, to be able to apply mathematics in real-world settings, and to think critically about mathematical results.

- Students need these skills in their other college courses, and in their jobs.
HOW DO WE DEVELOP CRITICAL THINKING SKILLS IN OUR STUDENTS?

• There are many type of questions we can ask that help students develop critical thinking skills.
SIMPLIFY VS. SOLVE

- Student Work:
  Simplify:

\[4x + 2(3x - 1)\]
\[4x + 6x - 2 = 0\]
\[10x = 2\]
\[x = \frac{1}{5}\]
• How about re-framing what we ask our beginning students, forcing them to think about how to answer the question.

• Example: \(4x + 2(3x - 1)\)

| a. Is this an expression or an equation? Explain. | b. Simplify if it is an expression. Solve if it is an equation. |
• One way to have our students think critically about mathematics is to have them look for common student errors in other’s work.

• **Example:** A student was asked to find the product and simplify. Explain what the student did wrong. Then simplify the expression correctly.

\[(t + 3)^2 = t^2 + 9\]
INTERPRETING INTERCEPTS

**Example:** Let \( D = 0.28t + 5.95 \) be the percentage of adults aged 18 years and older in the U.S. who have been diagnosed with diabetes, \( t \) years since 2000.

a. Find the vertical intercept. Explain its meaning in the context of the given situation.

b. Find the horizontal intercept. Explain its meaning in the context of the given situation.
• Answers:

a. The vertical intercept is (0, 5.95). 5.95% of adults aged 18 or older in the U.S. were diagnosed with diabetes in the year 2000. The vertical intercept makes sense in the context of the percentage of U.S. adults with diabetes.

b. The horizontal intercept is (-21.25, 0). The interpretation is that in 1979, no adults in the U.S. had been diagnosed with diabetes. This does not make sense in the context of the percentage of U.S. adults with diabetes.
INTERPRETING SLOPE

**Example**: Let $H(t) = -9t + 246.5$ be the average number of hours per year Americans spend listening to recorded music, $t$ years since 2000. Find the slope, and explain its meaning in the context of the problem.

**Answer**: The average number of hours that Americans spend listening to recorded music is declining at the rate of 9 hours per year.
The number of volunteers in the United States can be modeled by $V(t) = 1.3t + 51.6$ where $V(t)$ is the number of volunteers in the United States in millions $t$ years since 2000.

a. What is the $t$-intercept for this model and what does it represent in this context?

-39.7 is the number of volunteers in the United States since 2000.
WHAT DID THE STUDENT DO WRONG?

b. Find $V(10)$ and explain its meaning in this context.

c. What is the slope of this model and what does it represent in this context?
WHAT TYPE OF EQUATION IS THIS?

- One way to make students think critically about what technique they should use to solve an equation is to ask them what type of equation it is.
- Example: Classify each of the following equations as linear, quadratic, exponential, logarithmic, rational, radical or other.

<table>
<thead>
<tr>
<th>a. $\sqrt{3x - 4} = 2x + 1$</th>
<th>b. $\ln(x - 4) = 7$</th>
<th>c. $-3(x - 2) + 4 = 2x^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>d. $2^{x+1} = 8$</td>
<td>e. $5(3x - 1) + 4 = 2x - 7$</td>
<td>f. $\frac{3x^2 + 1}{5x - 2} = 6$</td>
</tr>
</tbody>
</table>
# Equation-Solving Toolbox

## Linear Equations

<table>
<thead>
<tr>
<th>Name</th>
<th>Users</th>
<th>Example</th>
<th>Where to Find It</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distributive Property</td>
<td>Use when an equation contains grouping symbols.</td>
<td>$3x + 4y = 5$</td>
<td>Section 1.1 and Appendix A</td>
</tr>
<tr>
<td>Addition Property of Equality</td>
<td>Use to isolate a term.</td>
<td>$x + 7 = 20$</td>
<td>Section 1.1 and Appendix A</td>
</tr>
<tr>
<td>Multiplication Property of Equality</td>
<td>Use to isolate a variable.</td>
<td>$5x = 13$</td>
<td>Section 1.1 and Appendix A</td>
</tr>
</tbody>
</table>

## Systems of Equations

<table>
<thead>
<tr>
<th>Method</th>
<th>Use when a variable is isolated or can be easily isolated.</th>
<th>Example</th>
<th>Where to Find It</th>
</tr>
</thead>
<tbody>
<tr>
<td>Substitution Method</td>
<td></td>
<td>$y = 4x + 8$</td>
<td>Section 2.2</td>
</tr>
<tr>
<td>Elimination Method</td>
<td></td>
<td>$2x + 4y = 30$</td>
<td>Section 2.3</td>
</tr>
</tbody>
</table>

## Quadratic Equations

<table>
<thead>
<tr>
<th>Name</th>
<th>Use when there is a squared term but no linear term.</th>
<th>Example</th>
<th>Where to Find It</th>
</tr>
</thead>
<tbody>
<tr>
<td>Square Root Property</td>
<td></td>
<td>$3x^2 - 4x + 5 = 0$</td>
<td>Section 4.4</td>
</tr>
<tr>
<td>Completing the Square</td>
<td></td>
<td>$x^2 + 4x + 4 = 0$</td>
<td>Section 4.4</td>
</tr>
<tr>
<td>Factoring</td>
<td></td>
<td>$x^2 + 3x + 1 = 0$</td>
<td>Sections 3.4, 3.5, and 4.5</td>
</tr>
<tr>
<td>Quadratic Formula</td>
<td></td>
<td>$13x^2 + 42x - 8 = 0$</td>
<td>Section 4.6</td>
</tr>
</tbody>
</table>

## Exponential Equations

<table>
<thead>
<tr>
<th>Name</th>
<th>Use when the equation has a variable in the exponent. Isolate the base and exponential part first.</th>
<th>Example</th>
<th>Where to Find It</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rewrite in logarithmic form</td>
<td></td>
<td>$5^x = 20 = 100$</td>
<td>Sections 6.2 and 6.5</td>
</tr>
</tbody>
</table>

## Logarithmic Equations

<table>
<thead>
<tr>
<th>Name</th>
<th>Use when the equation contains a logarithm. Isolate the logarithm first.</th>
<th>Example</th>
<th>Where to Find It</th>
</tr>
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<tbody>
<tr>
<td>Rewrite in exponential form</td>
<td></td>
<td>$\log_2(x + 2) = 4$</td>
<td>Sections 6.2 and 6.6</td>
</tr>
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</table>

## Rational Equations

<table>
<thead>
<tr>
<th>Name</th>
<th>Use when the equation contains a rational expression. Which the answers that cause denominators to be zero are therefore extraneous.</th>
<th>Example</th>
<th>Where to Find It</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiply both sides by the least common denominator</td>
<td></td>
<td>$\frac{3}{x+2} - 4 = \frac{2}{x-1}$</td>
<td>Section 7.5</td>
</tr>
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</table>
WHAT TYPE OF MODEL WILL FIT?
Applications

- The price of dental services are given in the table.

<table>
<thead>
<tr>
<th>Year</th>
<th>Price Index ($)</th>
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<tbody>
<tr>
<td>1960</td>
<td>25.2</td>
</tr>
<tr>
<td>1970</td>
<td>45.7</td>
</tr>
<tr>
<td>1980</td>
<td>82.4</td>
</tr>
<tr>
<td>1985</td>
<td>110.2</td>
</tr>
<tr>
<td>1990</td>
<td>149.7</td>
</tr>
<tr>
<td>1995</td>
<td>202.3</td>
</tr>
<tr>
<td>2000</td>
<td>270.9</td>
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Find a model for the data.
APPLICATIONS

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<td>2000</td>
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Find a model for the data. $t$ years since 1960

$D(t)$ is the price of dental services in dollars.
APPLICATIONS

- Quadratic Option

\[ D(t) = 0.169t^2 - 0.873t + 29.540 \]
\[ r^2 \approx 0.996 \]

- Exponential Option

\[ D(t) = 25.168(1.061)^t \]
\[ r^2 \approx 0.99997 \]
\[ r \approx 0.999987 \]
APPLICATIONS

- Quadratic Option
- Exponential Option

- Domain for these two model options may be different based on the end behavior of the models.
- Growth rate for the exponential model could be discussed with information regarding inflation.
U.S. WIND POWER CAPACITY

Source: http://www.thewindpower.net
CREATING AN EXPONENTIAL MODEL

• An exponential model can be generated by students using the form of an exponential function:
  \[ y = a \times b^x \]

• Select two points to write two equations using the form:
  \[ y = a \times b^x \]

• Divide the two equations and eliminate \( a \). Solve for \( b \).

• Substitute the value of \( b \) into the general form \( y = a \times b^x \).

• Substitute one of the points and solve for \( a \).

• Write the equation of the model.

• Check the model by graphing it on the scatterplot.
Find an equation that models the data.
Let $C(t)$ represent the U.S. wind power capacity in MW (megawatts) $t$ years since 1990.
Letting students select two points and finding the exponential model by hand:

\[ C(t) = 209(1.30)^t \]

Using the regression feature of the graphing calculator:

\[ C(t) = 247.63(1.28)^t \]
Questions for an exponential model:
1. What is the growth rate for this model? Explain the meaning of the growth rate in terms of the problem situation.
2. Find the vertical intercept of this model. Explain its meaning in terms of the problem situation.
3. What is a reasonable domain and range for this model?
WRAP-UP

- Questions?
- Suggestions?