Lend Me Your Ears:
How to Teach Your Principles Students About Loans

James A. Hornsten
Northwestern University

NETA – Dallas
November 2015

- The Loan Formula
- Why Loans in Principles?
- Loanable Funds & Life Cycle
- Financial Intermediation
- Interest Rates

- Time Value of Money
- Projects & NPV Rule
- Annuities
- Loans
- Applications
Today’s Takeaway: The Loan Formula

• One formula related the amount borrowed (B), the interest rate (r), the number of repayment periods (T), and the monthly payment (X).

• Given any three, we can find the fourth (B or X most easily).

• Intuitively, the \( \text{PV[Amount Borrowed]} = \text{PV[Annuity of Payments]} \)

• EXAMPLE: You want $200,000 today to buy a house. Your credit history enables you to get a standard, 30-year, fixed-rate mortgage with a 6% rate (APR, compounded monthly). Your monthly payment will be $1199.10.

\[
B = X \left[ \frac{1}{r} - \frac{1}{r(1+r)^T} \right] \Rightarrow 200,000 = X \left[ \frac{1}{0.005} - \frac{1}{0.005(1.005)^{360}} \right] \Rightarrow X = 1199.10 \approx 1200
\]

• INSIGHTS: If I want to borrow more to buy a pricier house, I could make a larger monthly payment, lower my discount rate (improve my credit rating), or pay it off over more months.

• If set up properly, a discussion of loans can shed light on important topics, demonstrate ECON’s value added, and help beginning students draw connections between numerous topics in macro & micro.
DISCLAIMER

• Using typical standards of a traditional PowerPoint presentation, the following slides are rather poor – too many distracting graphics & colors, complete sentences instead of phrases, and too many ideas per slide. Sorry. 😊
  ▪ I tend to use slides like these as class notes, and encourage students to bring the PDF to class and focus on listening and participating rather than racing to write down everything.

• Thus, please consider these slides as a set of suggestions about topics to include, a logical ordering, real world examples, things to (de-)emphasize or assume away, time to spend on each building block, etc.

So, Where Could Loans Fit Into Macro & Micro?
### Tesla’s Titan | Gigafactory to dwarf other U.S. battery plants in scale

Experts say Tesla’s plan carries big risks, but it may have no choice if it is to meet high sales goals.

<table>
<thead>
<tr>
<th>Select U.S. battery factories</th>
<th>Size</th>
<th>Employees</th>
<th>Capacity in gigawatt hours of battery produced in a year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed Tesla factory</td>
<td>10 million square feet</td>
<td>6,500</td>
<td>35.0</td>
</tr>
<tr>
<td>Site yet to be determined</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Under construction in Reno, NV</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LG Chem factory</td>
<td>600,000</td>
<td>125</td>
<td>1.0</td>
</tr>
<tr>
<td>Holland, Mich.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nissan Battery Factory</td>
<td>475,000</td>
<td>300</td>
<td>4.8</td>
</tr>
<tr>
<td>Smyrna, Tenn.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A123 Battery Factory</td>
<td>291,000</td>
<td>400</td>
<td>0.6</td>
</tr>
<tr>
<td>Livonia, Mich.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**50 GWh in annual battery production by 2020**
- Enough for 500,000 Tesla cars
- Powered by renewable energy
- Net zero energy factory

---

**Tesla’s Battery Gigafactory**


Markets for Houses and Other Real Estate

In 2007, Credit Suisse achieved something of a coup in what was then a much smaller, less mainstream financial blogosphere.

Analysts at the bank produced the following chart, which quickly (and uniquely) went viral, appearing on Calculated Risk and a slew of other housing and financial blogs. The chart even made a cameo appearance in the pages of an IMF report on ‘risks to global financial stability’.

Credit Suisse’s famous chart
http://ftalphaville.ft.com/2010/03/02/162856/
coming-soon-1000bn-resetting-recasting-us-arms/
Higher Education as an Investment in Human Capital

## The rewarding and the ruinous

Total cost of a degree*, 2013, $’000

<table>
<thead>
<tr>
<th>College</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>University of Virginia</td>
<td>50</td>
</tr>
<tr>
<td>Georgia Tech</td>
<td>50</td>
</tr>
<tr>
<td>Harvard</td>
<td>50</td>
</tr>
<tr>
<td>William and Mary</td>
<td>50</td>
</tr>
<tr>
<td>University of Washington</td>
<td>50</td>
</tr>
<tr>
<td>Stanford</td>
<td>50</td>
</tr>
<tr>
<td>MIT</td>
<td>50</td>
</tr>
<tr>
<td>U.C. Berkeley</td>
<td>50</td>
</tr>
<tr>
<td>Caltech</td>
<td>50</td>
</tr>
<tr>
<td>Dartmouth</td>
<td>50</td>
</tr>
<tr>
<td>Yale</td>
<td>50</td>
</tr>
<tr>
<td>Princeton</td>
<td>50</td>
</tr>
<tr>
<td>Purdue</td>
<td>50</td>
</tr>
</tbody>
</table>

Annual return over 20 years†, %

<table>
<thead>
<tr>
<th>College</th>
<th>Annual Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>University of Virginia</td>
<td>17.6</td>
</tr>
<tr>
<td>Georgia Tech</td>
<td>17.1</td>
</tr>
<tr>
<td>Harvard</td>
<td>15.1</td>
</tr>
<tr>
<td>William and Mary</td>
<td>14.8</td>
</tr>
<tr>
<td>University of Washington</td>
<td>14.8</td>
</tr>
<tr>
<td>Stanford</td>
<td>14.2</td>
</tr>
<tr>
<td>MIT</td>
<td>13.9</td>
</tr>
<tr>
<td>U.C. Berkeley</td>
<td>13.5</td>
</tr>
<tr>
<td>Caltech</td>
<td>13.3</td>
</tr>
<tr>
<td>Dartmouth</td>
<td>13.3</td>
</tr>
<tr>
<td>Yale</td>
<td>13.3</td>
</tr>
<tr>
<td>Princeton</td>
<td>13.2</td>
</tr>
<tr>
<td>Purdue</td>
<td>13.2</td>
</tr>
</tbody>
</table>

Source: PayScale

*After financial aid
†Earnings minus cost of college and earnings of a typical high-school graduate

20-Year Treasury Bill 3.4

<table>
<thead>
<tr>
<th>College</th>
<th>20-Year Treasury Bill</th>
</tr>
</thead>
<tbody>
<tr>
<td>College of New Rochelle</td>
<td>1.0</td>
</tr>
<tr>
<td>Morehead State University</td>
<td>0.6</td>
</tr>
<tr>
<td>Adams State College</td>
<td>0.5</td>
</tr>
<tr>
<td>Ashland University</td>
<td>-0.3</td>
</tr>
<tr>
<td>University of North Carolina at Asheville</td>
<td>-0.5</td>
</tr>
<tr>
<td>Faulkner University</td>
<td>-0.5</td>
</tr>
<tr>
<td>Ringling College of Art and Design</td>
<td>-0.5</td>
</tr>
<tr>
<td>Meredith College</td>
<td>-1.0</td>
</tr>
<tr>
<td>Maryland Institute College of Art</td>
<td>-1.1</td>
</tr>
<tr>
<td>Goshen College</td>
<td>-1.2</td>
</tr>
<tr>
<td>University of Montevallo</td>
<td>-3.2</td>
</tr>
<tr>
<td>College of the Ozarks</td>
<td>-3.3</td>
</tr>
<tr>
<td>Bluefield College</td>
<td>-4.6</td>
</tr>
<tr>
<td>Savannah State University</td>
<td>-6.9</td>
</tr>
<tr>
<td>Fayetteville State University</td>
<td>-10.6</td>
</tr>
<tr>
<td>Shaw University</td>
<td>-10.6</td>
</tr>
</tbody>
</table>
Should Government Bail Out Firms That Made Large Bets on Mortgage-Backed Securities?
Should The Fed Tinker with Interest Rates?
The Big Picture of Business:
A Systematic Series of Interdependent Steps

Was your venture successful and worth continuing (10)? It depends on the profit you generated by selling to customers (9) who responded to your pricing and advertising (8) of a delivered/distributed (7) product made somewhere & somehow (6) from inputs (5) purchased using capital obtained from lenders or investors (4) who were impressed by your presentation (3) of a forward-looking business plan (2) that fleshed out the details of a compelling business idea (1)!

This process applies to makers of lemonade, films, hybrid cars, software and legal advice.
Several Places Loans Could Fit Into Macro & Micro

• In Keynesian aggregate demand, \( Y = C + I + G + X – IM \), where business sector spending, or **Investment**, is prone to waves of optimism and pessimism, or “*animal spirits,*” like a herd of skittish zebras around a watering hole.
  ▪ Firms borrow $$$ to finance major projects (Elon Musk & Tesla’s Giga-Factory)
  ▪ Households/Individuals borrow for residential construction (housing) and/or to build productivity-enhancing human capital (college!)

• **Monetary Policy:** The Fed injects money into the system to lower interest rates, inspire lending and subsequent spending

• **Fiscal Policy:** Government spending may be financed by Treasury bonds, which are fancy versions of loans

• **Micro analysis of markets** for big-ticket items such as housing, autos, higher education, capital inputs
What is Finance? Why Study It?

- Most definitions address management of money, credit, investments.
- Finance generally means “how to pay for stuff” in a variety of contexts:
  - **Public Finance**
    - The US government uses income and FICA taxes to pay for national defense, Social Security, Medicare, and Medicaid. The State of Illinois uses sales and income taxes to pay for state highways and higher education (U of I, NIU). The City of Evanston uses property taxes to pay for the police department, fire department, and Evanston Township High School.
  - **Corporate or Business Finance**
    - Large firm such as General Motors decide which cars to build (Yes to Chevy Camaro & Cadillac CTS, but No to Geo, Saturn, Oldsmobile, Hummer, Pontiac), whether to build them in Mexico or the Motor City, whether to raise $ with stocks or bonds (or government loans), whether to issue dividends or repurchase shares, etc.
  - **Personal Finance**
    - You might lend $ through a bank deposit or buying an Illinois municipal bond; you might borrow to pay for a car, condo, or kid’s education; you might save for retirement through an individual retirement account

Common insights about borrowing by govt (macro), firm (micro), and household (micro). Large value added for 20 year-olds preparing to make large financial decisions.
Recent Financial Economic History

• The High-Tech Stock Market Bubble (late 1990s)
  ▪ The Internet and Dot-Coms
• Financial Contagion & International Capital Markets
  ▪ Latin America (esp. Mexico’s trouble with peso)
  ▪ SE Asia (esp. Thailand’s trouble with baht)
  ▪ Russia (default)
  ▪ Long Term Capital Management (T bet on Treasury spread)
• The Housing Bubble
  ▪ Low interest rates, lax lending standards, securitization (MBS, CDOs)
• The Financial Crisis and Credit Crunch
  ▪ Federal Bailouts of Bear Stearns, Fannie Mae, Freddie Mac, AIG, and GM
  ▪ Concerns about mortgage market: Was it too easy to borrow too much?
  ▪ The Troubled Asset Relief Program (TARP)
  ▪ Continuing weakness in the housing market; millions of underwater mortgages, a glut of foreclosures and short sales, and difficulty for many to get loans
  ▪ Interest rates have remained low; when will the Fed raise rates?

Story of $$$ moving b/t assets: tech stocks, emerging mkts, T-bills, real estate & MBS
THE LIFE CYCLE
Are you a supplier or demander in the market for loanable funds?

Are you more likely to be a saver or borrower at the following ages?

- 0 - 18 (lemonade stand)
- 18 - 22 (college, summer job)
- 20s (1st big job, vehicle, grad school, live with parents 😊, college debt)
- 30s (spouse, house, kids)
- 40s (teenagers)
- 50s (college tuition)
- 60s (grandkids)
- 70s and beyond (retirement)

We’re likely to be on different sides of this market at different life stages
College Lending Spree

In 2010, the federal government expanded its role in the student-loan market, making only direct loans instead of loan guarantees. Preliminary data show that in academic year 2011-12, it issued 93% of all student loans.

New student loans  In constant 2011 dollars

$120 billion

Debt outstanding  By type of consumer loan

$1.0 trillion

Includes bank, state and institutional loans  Preliminary  Note: New-loan data are for academic years that end in the year labeled

Sources: College Board (new loans); Federal Reserve Bank of New York (debt outstanding)

The Wall Street Journal
## More Debt Than They Realize

Many college students have taken on larger loans than they understand, a study finds. The misunderstanding is similar across two- and four-year colleges.

<table>
<thead>
<tr>
<th></th>
<th>Public four-year</th>
<th>Private non-profit four-year</th>
<th>Two-year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overestimate</td>
<td>22</td>
<td>27</td>
<td>22</td>
</tr>
<tr>
<td>Correctly estimate</td>
<td>24</td>
<td>23</td>
<td>29</td>
</tr>
<tr>
<td>Underestimate</td>
<td>54%</td>
<td>50%</td>
<td>49%</td>
</tr>
</tbody>
</table>

Students who estimated loan levels within 10 percent are considered to have done so correctly.

Source: Elizabeth Akers and Matthew Chingos, Brookings Institution
FINANCIAL INTERMEDIATION

**Playing Matchmaker**

- The primary role of the credit markets is to facilitate the transfer of money from *savers & lenders* (people who have extra money) to *borrowers* (people who need extra money). This act of serving as a middleman to create a better market for loanable funds is known as *financial intermediation*.

- For example, a *commercial bank* provides a safe-looking place (with lots of bricks, pillars, a massive metal vault door, security cameras and guards) to attract deposits which it then uses to make loans. The bank tries to borrow low and lend high: it may offer to pay 1% interest on a savings account and then charge 6% for a consumer loan (for an auto or house).

- It is EXTREMELY important that lenders are willing to lend so that borrowers may borrow; our modern economy relies heavily on credit (from Latin *credere*, to believe) and credibility.
The Return-Seeking Investor

- People with extra money usually want to spend it eventually on kids, houses, cars, toys, colleges, and/or retirement, but are willing to postpone its use. They invest in a variety of asset classes an attempt to make it grow. So do universities, pension plans, insurance companies, firms and governments.

  - STOCKS (equity) represent partial ownership in companies
  - BONDS (debt, fixed income securities) are loans, usually to firms or govts
  - MONEY refers to investments that are very liquid, i.e., can be converted quickly/easily/cheaply to cash. Examples: not just cash, but also bank certificates of deposit, safe short-term government bonds.
  - OTHER ASSETS include gold, art, wine, baseball cards, taxicab medallions

- On any investment, we can compute a rate of return to assess performance. A typical investment involves putting some money in today and hoping to get more back tomorrow, whether in the form of interest payments, dividends, or just selling “it” at a higher price than one paid for “it.” Intuitively, the way to compute return is to divide “got out - put in” by “put in.”
An Interest Rate Primer

• Conceptually, a rate of return compensates an investor (including lenders) for waiting and worrying
  
  - **WAITING**: If my dollars are currently tied up in ANY investment, it means that I cannot use them for other purposes, say, buying ice cream. I am impatient and I would like ice cream now, so I need to be convinced to forgo it. Temporarily.
    ⊢ Cash is convenient for unexpected bills, emergencies, & opportunities.
    ⊢ A kid prefers 1 cookie now to MANY later b/c very impatient
  
  - **WORRYING**: Economists define *risk* as “uncertainty that matters because of adverse consequence.” [Cf. Uncertainty over the contents of a wrapped birthday gift.] There are many types of risk with special names, but they’re all just different reasons to stay up at night and second-guess your investments.

• We use \( r \) (“little r”) to denote return and refer to it somewhat synonymously as a rate, discount rate, rate of return, return, or interest rate.

**We prefer to have dollars “now, for certain” to “later, maybe”**
Suppose there are many identical borrowers (with similar backgrounds and credit ratings) interested in renting somebody else’s money for five years in order to buy identical mid-size sedans. There are many identical lenders interested in lending $30,000.

Think of an interest rate as the price of renting somebody’s money

- If the interest rate were too high, there wd be too many lenders and not enough borrowers. If the price were too high, there wd be a surplus of loanable funds.
- If the interest rate were too low, there wd be too few lenders and too many borrowers. If the price were too low, there wd be a shortage of loanable funds.
- If the interest rate were just right, the borrowers and lenders wd just balance, and there wd be an equilibrium in the market for loanable funds.

Thus, the interest rate for a 5-year auto loan is partially generated by potential borrowers (car shoppers, the demand side of this mkt) and potential lenders (banks, the supply side). It’s basically the price the market decides is appropriate.

Insight: We can look in the Money & Investing section of the Wall Street Journal to find a number of useful current interest rates. These are just prices that have been generated by S&D forces in various credit markets.
Examples of Interest Rates (October 2014)

Interpret these rates as prices of using loanable fund for various purposes.

---

**Interest Rates**

### Treasury Yields

<table>
<thead>
<tr>
<th>U.S. 2 Year</th>
<th>U.S. 10 Year</th>
<th>U.S. 30 Year</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="https://via.placeholder.com/150" alt="Graph" /></td>
<td><img src="https://via.placeholder.com/150" alt="Graph" /></td>
<td><img src="https://via.placeholder.com/150" alt="Graph" /></td>
</tr>
</tbody>
</table>

### Consumer Money Rates

<table>
<thead>
<tr>
<th>Interest Rate</th>
<th>YIELD/RATE %</th>
<th>52-WEEK</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LAST</td>
<td>WK AGO</td>
</tr>
<tr>
<td>Federal-funds rate target</td>
<td>0-0.25</td>
<td>0-0.25</td>
</tr>
<tr>
<td>Prime rate*</td>
<td>3.25</td>
<td>3.25</td>
</tr>
<tr>
<td>Money market, annual yield</td>
<td>0.42</td>
<td>0.42</td>
</tr>
<tr>
<td>Five-year CD, annual yield</td>
<td>1.51</td>
<td>1.52</td>
</tr>
<tr>
<td>30-year mortgage, fixed</td>
<td>4.01</td>
<td>4.07</td>
</tr>
<tr>
<td>15-year mortgage, fixed</td>
<td>3.16</td>
<td>3.23</td>
</tr>
<tr>
<td>Jumbo mortgages, $417,000-plus</td>
<td>4.47</td>
<td>4.26</td>
</tr>
<tr>
<td>Five-year adj mortgage (ARM)</td>
<td>3.64</td>
<td>3.53</td>
</tr>
<tr>
<td>New-car loan, 48-month</td>
<td>3.22</td>
<td>3.22</td>
</tr>
</tbody>
</table>

**10/17/2014**

**CHANGE IN PCT. PTS**

<table>
<thead>
<tr>
<th></th>
<th>52-WK</th>
<th>3-YR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>0.01</td>
<td>-0.09</td>
</tr>
<tr>
<td></td>
<td>0.16</td>
<td>-0.05</td>
</tr>
<tr>
<td></td>
<td>-0.33</td>
<td>-0.35</td>
</tr>
<tr>
<td></td>
<td>-0.27</td>
<td>-0.49</td>
</tr>
<tr>
<td></td>
<td>-0.09</td>
<td>-0.54</td>
</tr>
<tr>
<td></td>
<td>-0.04</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td>0.26</td>
<td>-0.92</td>
</tr>
</tbody>
</table>

**Source:** [http://online.wsj.com/mdc/page/marketsdata.html?mod=WSJ_topnav_marketdata_main](http://online.wsj.com/mdc/page/marketsdata.html?mod=WSJ_topnav_marketdata_main)
**Interest Rates**

*November 2015*

We observe that rates change over time and then tend to move together (though not in lockstep).

### Treasury Yields

<table>
<thead>
<tr>
<th>U.S.2 Year</th>
<th>U.S.10 Year</th>
<th>U.S.30 Year</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.10</td>
<td>3.05</td>
</tr>
</tbody>
</table>

### Consumer Money Rates

<table>
<thead>
<tr>
<th>Interest Rate</th>
<th>52-WEEK</th>
<th>3-YR</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>YIELD/RATE %</strong></td>
<td>HI</td>
<td>LO</td>
</tr>
<tr>
<td>Federal-funds rate target</td>
<td>0.025</td>
<td>0.025</td>
</tr>
<tr>
<td>Prime rate*</td>
<td>3.25</td>
<td>3.25</td>
</tr>
<tr>
<td>Money market, annual yield</td>
<td>0.28</td>
<td>0.30</td>
</tr>
<tr>
<td>Five-year CD, annual yield</td>
<td>1.42</td>
<td>1.48</td>
</tr>
<tr>
<td>30-year mortgage, fixed</td>
<td>3.86</td>
<td>3.76</td>
</tr>
<tr>
<td>15-year mortgage, fixed</td>
<td>3.06</td>
<td>2.97</td>
</tr>
<tr>
<td>Jumbo mortgages, $417,000-plus</td>
<td>4.47</td>
<td>4.35</td>
</tr>
<tr>
<td>Five-year adj mortgage (ARM)</td>
<td>3.52</td>
<td>3.49</td>
</tr>
<tr>
<td>New-car loan, 48-month</td>
<td>3.11</td>
<td>3.14</td>
</tr>
</tbody>
</table>

**11/04/2015**

*Federal-funds, prime rate updated as needed late evening; Libor updated late afternoon; all other rates updated by 7 p.m. ET.*

Interest Rates Generally Move Together

*Marching Out of Sync*

Different types of interest rates don’t usually move in exact sync. Several rates for the past 10 years:

Sources: Ryan ALM (10-year yields), Bankrate (mortgage and CDs), WSJ Market Data Group

Michael Pollock, “How Your Rates Will Move When the Fed Does,” WSJ, 6/7/15
And now, a bit of time travel.

Back and forth through time.  
For piles of money.  
Not you. (Sorry)
Consider a Savings Account that Pays 10%

You deposit $100 at your local bank, which promises to pay 10% annual compound interest. Synonyms: interest rate, growth rate, discount rate, "little r". Here, \( r = 0.1 \)

Your initial deposit is called the principal
Annual \( \Rightarrow \) pays interest once per year
Compound \( \Rightarrow \) pays interest on the current balance, not just the original deposit

At the end of one year you will earn $10 interest, or 10% of $100
MATH: \( $100(1 + r) = $100(1 + 0.1) = $100(1.1) = $110 = $100 \text{ principal} + $10 \text{ interest} \)

At the end of two years, you will earn $11 interest, or 10% of $110
MATH: \( $110(1.1) = $100(1.1)(1.1) = $100(1.1)^2 = $121 \)

NOTE: $121 = $100 \text{ principal} + $10 \text{ interest} + $10 \text{ interest} + $1 \text{ interest on interest}

INSIGHT: The interest on interest can be substantial if you invest for a long time.
At the end of ten years, $100(1.1)^{10} = $100(2.5937) = $259.37, not just $200.

After fifty years, $100(1.1)^{50} = $100(117.3909) = $11,739.09, not just $(10)(12)(50) = $6,000

Sadly, it may be hard to find a return of 10% without incurring a fair amount of risk.

10% is a good rate for illustration, though not so realistic nowadays...
The Time Value Of Money

• Armed with the knowledge that interest rates adjust for risk, we use them to move piles of money across time. And if we can handle one pile, we can handle sets of related piles by breaking them into manageable parts.

• We just saw that a $100 saving account deposit grows to $110 in one year. We refer to $100 as the **present value (PV)** and $110 as the **future value (FV)** in one year. Likewise, $100 is the PV and $259.37 is the FV in ten years for \( r = 0.1 \).

• Moving money piles forward in time is called **compounding**, while moving them backward in time is called **discounting**. The “little r” used in the denominator when discounting is known as the **discount rate**.

(Present Value)\( \times (1 + r) = \) Future Value
($100)(1.1) = $110 \text{ in one year.}$
($100)(1.1)(1.1) = $121 \text{ in two years.}$
($100)(1.1)^{10} = $259.37 \text{ in ten years.}$

Compounding!

Present Value = \( \frac{\text{Future Value}}{1+r} \)

\[ $100 = \frac{$110}{1+.10} \]
\[ $100 = \frac{$121}{(1.1)^2} \]
\[ $100 = \frac{$259.37}{(1.1)^{10}} \]

Discounting!
Before Proceeding, Be Sure You Understand This.

$100$ invested today at $10\%$ will grow to $259.39$ in ten years.  
$100(1.1)^{10} = 259.37$ or more generally, $PV(1+r)^T = FV$, where  
$PV=\text{Present Value}$, $r=\text{discount rate}$, $T=\#\text{periods}$, $FV=\text{future value}$  
Here, PV=100, $r=0.1$, $T=10$, and $FV=259.37$  
This is COMPOUNDING, or tracking how a pile of money from today  
GROWS over time as we move forward in time.

A $259.39$ pile of money to be received in ten years is worth  
$100$ today if the discount rate is $10\%$.  
$100 = \frac{259.37}{(1.1)^{10}}$ or more generally, $PV = \frac{FV}{(1+r)^T}$  
This is DISCOUNTING, or tracking how a pile of money from the  
future SHRINKS over time as we move backward in time.

**KEY INSIGHT:** ALL of our finance problems are based on moving  
piles of money backward or forward in time using these PV and FV formulas.

**Note that this is really just one formula written in two ways!**
A “Project” is a Set of Related Cash Flows

• We can move individual piles of money backwards in time by discounting.
• Now we want to analyze complex financial projects which involve multiple piles of money (or cash flows). Most interesting projects involve both cash inflows ($\text{IN}$) and outflows ($\text{OUT}$). E.g., many projects require an up-front initial investment, and then generate revenues at future dates.
• Here, a simple loan features one large withdrawal today (a cash inflow to a borrower), followed by a series of equal monthly payments (cash outflows from the borrower).
• The easiest way to handle a complex set of cash flows is to break them up into smaller, more manageable parts.
• In general, it would be useful to be able to understand a wide variety of projects, some of which feature uneven cash flows: both $\text{IN}$ and $\text{OUT}$ in various sizes.
• We want to compare apples to apples, not apples to oranges.
• E.g., $100$ in $2015$ is not the same as $100$ in $2025$.
• How do we find a common frame of reference?!
Making Decisions With Net Present Value

- **KEY**: All cash flows have today (time \(t=0\)) in common. Any cash flow occurring in the future could be expressed in today’s dollars by discounting. 😊
- If multiple cash flows are all expressed in today’s dollars, then we could add them.
- We convert all of the cash flows to their value at \(t=0\), the present value.
- **Net Present Value** = The sum of the present values of ALL of a project’s related cash flows, both positive and negative, large and small.
  - \(\text{NPV} = \text{PV}[\text{Cash Inflows, } $^\text{IN}] + \text{PV}[\text{Cash Outflows, } $^\text{OUT}]\)
- **DECISION RULE**: Compute NPV, and then examine its sign. If NPV<0, do not invest in this project. If NPV=0, you are indifferent. If NPV>0, invest!
- When we discount future cash flows (by dividing by a power of \(1+r\)), we are automatically adjusting them for our concerns about impatience and risk (or “waiting and worrying”) for that particular project.
- Different projects may use different discount rates if they have different risks!
  - ASSUME this away for now
- Is NPV>0? Investors prefer projects with NPV>0, for if not, the project is not creating value. Intuitively it would be nice to rent money at 2% and use it to generate a 5% return.
- If there are multiple projects, then which has largest NPV?
One Formula to Rule Them All

We will adapt one formula many times to analyze interesting and relevant finance problems. But before we tackle the math, let us consider some fundamentals of finance.

\[ NPV = \sum_{t=0}^{\infty} \frac{ECF_t}{(1 + r)^t} = \frac{ECF_0}{(1 + r)^0} + \frac{ECF_1}{(1 + r)^1} + \frac{ECF_2}{(1 + r)^2} + \ldots \]

\[ NPV = \text{Net Present Value} \]

\[ \sum \quad \text{Sigma notation: a summation of terms indexed by } t \]

\[ t = \text{A particular time period, such as } t = 0 \text{ or } t = 7 \]

\[ ECF_t = \text{The expected cash flow (a pile of money) at time } t \]

\[ r = \text{The discount rate, compensation for waiting & worrying} \]

\[ \frac{1}{(1 + r)^2} = \text{The term used to discount a pile of money two periods} \]

\[ \frac{ECF_2}{(1 + r)^2} = \text{The present value of the pile of money from period } t = 2 \]
The Mental Leap: Many Interesting “Projects”

• **Bonds:** Lend $ today, collect interest payments until maturity date, then collect face repayment

• **Shares:** Buy share today at $\text{P}^\text{BUY}$, collect dividends until you resell the share and collect the selling price, $\text{P}^\text{SELL}$

• **Car Loan:** You borrow Principal today, then make 60 equal monthly car payments over the next 5 years

• **Mortgage:** You borrow Principal today, then make 360 equal monthly house payment over the next 30 years.

• **Retirement Account:** You want to have $5m to withdraw when you retire at age 70, so you make 600 equal monthly deposits over the next 50 years

• **Copyright Term Extension:** How much wd a ©-holder be WTP to extend © protection by 20 years?

• **Product Recall Decision:** Does it make sense to recall a defective product or just deal with the lawsuits?

• **Cartel Behavior:** Should I stick with the long-run deal or backstab my co-conspirators for a short-run windfall?
Using NPV to Analyze Various “Projects”

\[ NPV = 0 = -\text{Initial Investment} + PV[\text{Cash Inflows}] \]
⇒ Maximum Initial Investment = \( PV[\text{Cash Inflows}] \)

\[ NPV = 0 = -P^{BOND} + PV[\text{Coupons}] + PV[\text{Face}] \]
⇒ \( P^{BOND} = PV[\text{Coupons}] + PV[\text{Face}] \)

\[ NPV = 0 = -P^{SHARE} + PV[\text{Dividends}] + PV[\text{Resale Price}] \]
⇒ \( P^{SHARE} = PV[\text{Dividends}] + PV[\text{Resale Price}] \)

\[ NPV = 0 = -\text{Loan Amount} + PV[\text{Monthly Payments}] \]
⇒ Loan Amount = \( PV[\text{Monthly Payments}] \)

\[ NPV = 0 = -PV[\text{Balance At Retirement}] + PV[\text{Deposits}] \]
⇒ \( PV[\text{Balance At Retirement}] = PV[\text{Deposits}] \)

KEY INSIGHT: If financial assets are fairly priced by reasonably competitive markets, then we expect normal rates of return and \( NPV=0 \). When \( NPV\neq0 \), we could think of the assets as being either over- or under-valued, which means there is a profitable opportunity for someone who can identify the pricing discrepancy and act on it. E.g., investors like Warren Buffett try to identify firms with strong fundamental indicators, buying their shares now at low prices and anticipating that they will rise substantially in value. If we believe that Wall Street is full of thousands of smart, connected, wealth-seeking people with clear incentives to dig up and trade upon valuable information, then for a starting point, we can reasonably assume that assets like bonds and shares of stock should be fairly priced, and these formulas for \( P^{BOND} \) and \( P^{SHARE} \) should be pretty accurate. Of course, all of them are based on discount rate \( r \), which is the subject of some controversy.

There are MANY ways one could use NPV to teach ECON or personal finance
PROJECT: Music Producer

• You are a music producer contemplating whether to sign a new artist to your record label. You anticipate the cash flows shown in the table below. (They would be much higher for an established artist.) Each time period is about 3 months (or one quarter).

• What info is needed to make this decision?

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Explanation</th>
<th>Cash Flow ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>t=0</td>
<td>Pay artist’s advance</td>
<td>-50K</td>
</tr>
<tr>
<td>t=1</td>
<td>Produce a single (e.g., music studio)</td>
<td>-20K</td>
</tr>
<tr>
<td>t=2</td>
<td>Market the single (e.g., signs, demos, video)</td>
<td>-250K</td>
</tr>
<tr>
<td>t=3, etc.</td>
<td>Sell the single; pay a total of 20% royalties from sales revenue to artist, song writer, etc.</td>
<td>???</td>
</tr>
</tbody>
</table>

An optional digression into financial projects before diving into loans
PROJECT: Music Producer

Ideally, you can forecast the discount rate and future cash flows (size, number, timing)

\[ NPV_{\text{PROJECT}} = \sum_{t=0}^{\infty} \frac{\text{Expected Cash Flow at time } t}{(1 + \text{Periodic Discount Rate})^t} \]

\[ NPV = -50 - \frac{20}{(1+r)} - \frac{250}{(1+r)^2} + \frac{0.8(Sales_3)}{(1+r)^3} + \frac{0.8(Sales_4)}{(1+r)^4} + ... + \frac{0.8(Sales_T)}{(1+r)^T} \]

CASE: Flop! \( r = .03, Sales_3 = 10, T = 3 \)

\[ NPV = -50 - \frac{20}{(1.03)} - \frac{250}{(1.03)^2} + \frac{0.8(10)}{(1.03)^3} \approx -50 - 19 - 236 + 7 = -297 \]

CASE: Hit! \( r = .03, Sales_3 = 1000, T = 3 \)

\[ NPV = -50 - \frac{20}{(1.03)} - \frac{250}{(1.03)^2} + \frac{0.8(1000)}{(1.03)^3} \approx -50 - 19 - 236 + 732 = 427 \]

CASE: Classic! \( r = .03, Sales_3 = Sales_4 = Sales_5 = 1000, T = 5 \)

\[ NPV = -50 - \frac{20}{(1.03)} - \frac{250}{(1.03)^2} + \frac{0.8(1000)}{(1.03)^3} + \frac{0.8(1000)}{(1.03)^4} + \frac{0.8(1000)}{(1.03)^5} \]

\[ \approx -50 - 19 - 236 + 732 + 711 + 690 = 1828 \]
PROJECT: Music Producer

For this music producer project the general equation seems to be

\[
NPV_{\text{PROJECT}} = \sum_{t=0}^{\infty} \frac{\text{Expected Cash Flow at time } t}{(1 + \text{Periodic Discount Rate})^t}
\]

\[
NPV = -\text{Advance} - \frac{\text{Studio}}{(1+r)} - \frac{\text{Marketing}}{(1+r)^2} + \sum_{t=3}^{\infty} \frac{(0.8)\text{Sales}_t}{(1+r)^t}
\]

When does this project become more lucrative?

Advance, Studio, and/or Marketing Cost falls
Discount rate falls (less risky for established artist with loyal fan base)
Sales Revenues are larger, last for more periods, start sooner

You could imagine a more complex situation in which there are
more expenses (e.g., marketing continues past \( t=2 \))
different types of sales & royalties (e.g., iTunes, Pandora, FM, TV ad)
different period lengths (spend 6 mos recording, only 2 marketing)
A Timeline for a Simple Loan of $1000

- Horizontal arrow from 0 (today!) to ending period (T or \( \infty \))
- Tick marks and numbers at ends of relevant periods
- Record cash flow numbers at correct period. $^{\text{IN}}$ are positive and $^{\text{OUT}}$ are negative.
- Write discount rate above each period (or just 1st period if there’s only one rate)
- **Would you prefer to be the borrower or lender here?**
- Find the NPV, the sum of the present values of the five cash flows

The relevant periodic discount rate is 10%

The last cash flow occurs at \( t=4 \), which we denote as terminal period, \( T \)

<table>
<thead>
<tr>
<th>Time (t)</th>
<th>Cash Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1000</td>
</tr>
<tr>
<td>1</td>
<td>+300</td>
</tr>
<tr>
<td>2</td>
<td>+300</td>
</tr>
<tr>
<td>3</td>
<td>+300</td>
</tr>
<tr>
<td>4</td>
<td>+300</td>
</tr>
</tbody>
</table>
This Loan Benefits the Borrower, But Harms the Lender

\[ NPV = PV[All \ Cash \ Flows] = PV[All \ Cash \ Inflows] + PV[All \ Cash \ Outflows] = \]

\[ CF_0 + \frac{CF_1}{1+r} + \frac{CF_2}{(1+r)^2} + \frac{CF_3}{(1+r)^3} + \frac{CF_4}{(1+r)^4} = 1000 + \frac{-300}{1.1} + \frac{-300}{(1.1)^2} + \frac{-300}{(1.1)^3} + \frac{-300}{(1.1)^4} \]

\approx 1000 - 272.73 - 247.93 - 225.39 - 204.90 \approx 49.04 > 0

Because NPV>0, borrower should accept this project (i.e., take out the loan).

The lender's NPV would be -49.04 < 0, so s/he should refuse this loan.

- A buyer and seller will only voluntarily trade if this is mutually beneficial (weakly).
  Similarly, a borrower and lender will only agree to voluntarily trade at a mutually
  beneficial interest rate. THUS, let’s assume that equilibrium forces in the financial
  markets drive the interest rate to a “fair” level at which NPV = 0.

- When NPV=0, PV[cash inflows] = PV[cash outflows], or in the context of a loan,
  PV[amount borrowed] = PV[series of all monthly payments]. Lenders prefer
  charging a high rate, and borrowers prefer paying a low rate, so NPV=0
  characterizes a loan with terms that are acceptable to both lenders and borrowers.

In short, a “fair” interest rate sets NPV = 0, so \[ PV[\$^{in}] = PV[\$^{out}] \]
NPV depends on the size and timing of each cash flow and the discount rate (risk!).

Small $r$ implies low risk, while high $r$ implies high risk.

<table>
<thead>
<tr>
<th>Discount Rate</th>
<th>0.10</th>
<th>0.09</th>
<th>0.08</th>
<th>0.07</th>
<th>0.075</th>
<th>0.077</th>
<th>0.0771</th>
<th>0.0772</th>
<th>0.07715</th>
<th>0.07714</th>
<th>0.077139</th>
<th>0.0771385</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period</td>
<td>Cash Flow</td>
<td>PV</td>
<td>PV</td>
<td>PV</td>
<td>PV</td>
<td>PV</td>
<td>PV</td>
<td>PV</td>
<td>PV</td>
<td>PV</td>
<td>PV</td>
<td>PV</td>
</tr>
<tr>
<td>0</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>1</td>
<td>-300</td>
<td>-272.73</td>
<td>-275.23</td>
<td>-277.78</td>
<td>-280.37</td>
<td>-279.07</td>
<td>-278.55</td>
<td>-278.53</td>
<td>-278.50</td>
<td>-278.51</td>
<td>-278.52</td>
<td>-278.52</td>
</tr>
<tr>
<td>2</td>
<td>-300</td>
<td>-247.93</td>
<td>-252.50</td>
<td>-257.20</td>
<td>-262.03</td>
<td>-259.60</td>
<td>-258.64</td>
<td>-258.59</td>
<td>-258.54</td>
<td>-258.56</td>
<td>-258.57</td>
<td>-258.57</td>
</tr>
<tr>
<td>3</td>
<td>-300</td>
<td>-225.39</td>
<td>-231.66</td>
<td>-238.15</td>
<td>-244.89</td>
<td>-241.49</td>
<td>-240.15</td>
<td>-240.08</td>
<td>-240.01</td>
<td>-240.05</td>
<td>-240.05</td>
<td>-240.05</td>
</tr>
<tr>
<td>4</td>
<td>-300</td>
<td>-204.90</td>
<td>-212.53</td>
<td>-220.51</td>
<td>-228.87</td>
<td>-224.64</td>
<td>-222.98</td>
<td>-222.89</td>
<td>-222.81</td>
<td>-222.85</td>
<td>-222.86</td>
<td>-222.86</td>
</tr>
</tbody>
</table>

NPV = 49.04 28.08 6.36 -16.16 -4.80 -0.31 -0.09 0.14 0.03 0.003 0.001 0.0001

We can find rate at which NPV=0 so borrower 👍 and lender 😞.

When $r$ is small (here, $r<0.0771385$), NPV<0 and project shd be rejected by borrower, who is less impatient and more likely to make the payments. When $r$ gets larger (reflecting more risk that borrower defaults and more impatience), we’ll find NPV>0, so borrower 😞 while lender 👍. When $r = 7.71385\%$, both borrower and sender are 😞.
Annual Percentage Rate (APR)

- For legal purposes, rates are typically expressed as an annual percentage rate, which is the product of the periodic rate and the number of periods.
- One might think of APR as a marketing gimmick since the actual rate one pays is generally higher. We’ll always use the effective periodic rate.
- E.g., 12% APR compounded monthly translates to 12.6825% effective annual rate!

12% APR compounded monthly

\[
\frac{12\% \text{APR}(\text{monthly})}{12 \text{ months}} = \frac{1\%}{\text{month}}, \text{ so the relevant period is one month}
\]

How much will a deposit grow in 12 month at 1% each month?

In 12 months, \( X \) grows to \( FV = X(1 + r)^T = X(1.01)^{12} = X(1.126825) \)

Thus, 12% APR(compounded monthly) = 12.6825% effective annual rate

The advertised rate is NOT what you actually pay!

THUS, we shd use 1%/month, NOT 12%/yr, when doing calculations

Whenever we are given an APR, we should convert it to an effective periodic rate
Annuities are Common; Use the Shortcut!

- An **annuity** is an evenly-spaced stream of regular payments
  - Annual retirement account deposits
  - Semi-annual bond coupons
  - Quarterly dividend checks
  - Monthly house or car payments
  - Weekly allowance
  - $3 coffee every morning

\[ C = \text{Regular cash flow ($IN$ or $OUT$)} \]
\[ r = \text{periodic discount rate} \]
\[ T = \# \text{ of periods} \]
\[ PV = \text{present value} \]

TIP: If there are 3 or more regular payments, it usually helps to use this annuity formula (a time-saving shortcut!). \( PV_{ANN} \) or \( PV_{ANNUITY} \) means the present value of an annuity, and it converts a series of regular payments to a single pile of money in the period before the first payment. So if you start repaying a loan at \( t=1 \), \( PV_{ANN} \) rests at \( t=0 \).

\[
PV_{ANNUITY} [C,r,T] = C \left[ \frac{1}{r} \frac{1}{r(1+r)^T} \right]
\]

If \( C = 10, \ r = .02, \text{ and } T = 40 \), then

\[
PV_{ANNUITY} = 10 \left[ \frac{1}{.02} - \frac{1}{.02(1.02)^{40}} \right]
\]

\[= 10[48.65623615] \]

\[ PV_{ANN} = \left( \frac{10}{.02} \right) \left[ 1 - \frac{1}{(1.02)^{40}} \right] \] is calculator-friendly, though we won't be using calculators on exams.
Our Loan Contains an Annuity of Repayments

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount rate ( r )</td>
<td>1000</td>
<td>-300</td>
<td>-300</td>
<td>-300</td>
</tr>
</tbody>
</table>

- The payments must be constant and regularly spaced. Here, there are 4 evenly spaced payments of $300.
- The annuity shortcut formula computes the PV of the annuity at the previous period. Here, the $300 repayments occur at times \( t=1,2,3,4 \), so the PV must be computed at \( t=0 \). (If payments were at \( t=5,6,7,8 \), then “PV_{ANN}” would rest at \( t=4 \))

\[
NPV = PV[\text{AmtBorrowed}] + PV[\text{Repayments}] = +1000 + (-300) \left[ \frac{1}{r} - \frac{1}{r(1+r)^4} \right],
\]

which is faster to write than

\[
+1000 + \frac{-300}{1+r} + \frac{-300}{(1+r)^2} + \frac{-300}{(1+r)^3} + \frac{-300}{(1+r)^4}
\]
A Solved Problem: Find Monthly Mortgage Payment

- You need $200,000 today to buy a car/house/diploma
- IDEA: \( PV[\text{Amount borrowed}] = PV[\text{Annuity of Payments}] \)
- Suppose you could borrow at a 6% rate (APR, compounded monthly).
- You get a standard, 30-year, fixed-rate mortgage
- Given the amount borrowed (\( B = $200,000 \)), the interest rate (6% APR/12mos) = 0.5%/mo = \( r \)), and the # of periods (\( T = 30\text{yrs} \times 12\text{mos/yr} = 360 \text{mos} \)), we can compute the monthly payment of about $1199.10
- In fact, if we are given any three of the four variables (\( AB, X, r, T \)), we can compute the fourth. You can use this when you go to get a loan!

\[
\begin{align*}
PV[\text{Large Withdrawal Today}] &= PV[\text{An Annuity of Small Deposits}] \\
\text{Amount Borrowed } B &= PV^{\text{ANNUITY}} \left[ \text{Payment } X, \#\text{Periods } T, \text{Interest Rate } r \right] \\
B &= X \left[ \frac{1}{r} - \frac{1}{r(1+r)^T} \right] \\
$200,000 &= X \left[ \frac{1}{0.005} - \frac{1}{0.005(1.005)^{360}} \right] \Rightarrow 1199.10
\end{align*}
\]
TINKERING: Change One Assumption, and Then Resolve the Familiar Problem

- We can also use this formula to determine how much one could comfortably borrow, the implications of changing interest rates, the importance of building a good credit record, etc.
- Benchmark: $B=200,000$, $r = 0.5\%/\text{mo}$, $T = 360 \text{ mos}$ implies $X = 1199.10$
- How much cd you borrow if you made 360 monthly payments of $1500$?
- How much cd you borrow if you made 480 monthly payments of $1200$?
- Suppose this borrower improved his/her credit score so much that s/he could qualify for a 3\% (APR, compounded monthly) rate. If $B=200,000$, and $T=360$, compute the new monthly payment, $X$. 

\[
B = 1500 \left[ \frac{1}{0.005} - \frac{1}{0.005(1.005)^{360}} \right] \implies B = \\
B = 1200 \left[ \frac{1}{0.005} - \frac{1}{0.005(1.005)^{480}} \right] \implies B = \\
\$200,000 = X \left[ \frac{1}{0.0025} - \frac{1}{0.0025(1.0025)^{360}} \right] \implies X =
\]
Old Exam Questions

4. SPA-MOBILE. Your latest entrepreneurial idea: Even better than going to the spa is having the spa come to you! Of course, this requires buying an unusual, $100,000 custom-built vehicle. You plan to buy this vehicle and pay it off with monthly payments over the next 5 years. A bit of research suggests that if you can improve your credit score, you can qualify for a 4.8% (APR, compounded monthly) loan rather than a 6.0% (APR, compounded monthly) loan. It turns out that the monthly payments for the 6.0% loan are about $1933. How much would you save each month if you could qualify for the 4.8% loan? Use the annuity formula in your answer.

With a loan, \( PV[\text{amount borrowed}] = PV[\text{amount repaid}] \). Convert the rate: 4.8% APR/12 months = 0.4%/month, or \( r = .004 \). Determine the # of periods: (5 years)(12 mos/yr) = 60 months = \( T \). Write out the general formula, and then plug in the correct numbers:

\[
AB = X \left[ \frac{1}{r} - \frac{1}{r(1+r)^T} \right] = 100,000 = X \left[ \frac{1}{.004} - \frac{1}{.004(1.004)^{60}} \right]
\]

and then solve this equation for \( X \).

If you would pay $1933 with a 6.0% loan, but only \( X \) with a 4.8% loan, then you wd save \( 1933 - X \)

2. GRAD STUDENT LOAN. Congrats! You will start a two-year grad school program to develop your human capital. To finance this you borrow $30,000 today (at \( t=0 \)) and $30,000 next year (at \( t=1 \)), and then pay it off over the next ten years with annual payments starting at time \( t=2 \). If your credit history allows you to borrow at a 6% annual rate, then show how to compute \( X \), your annual payment. For full credit, your answer should feature an annuity.

\[
NPV = 0 = PV[\text{withdrawals}] + PV[\text{payments}]
\]

\[
0 = -30,000 - \frac{30,000}{1.06} + \frac{X}{.06} \left[ \frac{1}{.06} - \frac{1}{.06(1.06)^{10}} \right]
\]

The challenging part is recognizing that our annuity formula is used in the period before the payments begin. Here, you apply the formula at \( t=1 \) for payments that start at \( t=2 \); thus, you discount the annuity blob only one period.
“But This Annuity Formula is Going to Scare My Students!”

• An alternative approach emphasizes financial insights:
  - The amount borrowed depends on the regular payment, interest rate, and number of periods.
  - The monthly payment depends on the amount borrowed, the interest rate, and number of periods.
  - A lower interest rate either allows one to borrow more OR to make lower payments OR to shorten the life of the loan.
    - The interest rate one pays depends on one’s credit history, one’s willingness/ability to shop around and negotiate, discrimination by lenders, and other forces affecting the S&D for such loans
    - THUS, 20 year-olds can start thinking about establishing good financial habits, avoiding bad debt (e.g., borrowing $$$ to buy a mountain of potato chips probably doesn’t enhance productivity or the ability to repay the loan), establishing a record of paying bills on time

We have an opportunity to plant some good ideas without being too preachy
One Could Also Use a Black Box (Hidden Formula) Approach

- Setting up a basic Excel spreadsheet gives them a chance to play around with the 4 variables.
- NOTE: The graphic shown below is not mine, but shows the basic idea. One could use Excel’s PMT function or just enter the loan formula.

\[
B = X \left[ \frac{1}{r} - \frac{1}{r(1+r)^T} \right]
\]

\[
\Rightarrow X = \frac{B}{\left[ \frac{1}{r} - \frac{1}{r(1+r)^T} \right]}
\]

where \( X \) is monthly payment, \( B \) is loan amount, \( r \) is monthly interest rate (APR/12), and \( T \) is # of monthly payments (from years x 12 payments/year).

We could teach students a little about loans without going through 40 technical slides!
APPLY: IRAs Focus on Retirement Date

- Once students understand loans, it’s not hard to switch gears and take about individual retirement accounts.
- Whereas a loan focuses attention on today (t=0), an IRA problem usually focused on the IRA balance on the retirement date, as if one retired and then promptly withdrew an entire 401(k) or 403(b), or at least could. 😊
- Loan: \( PV[\text{Large Withdrawal at } t=0] = PV[\text{Annuity of Small Deposits at times } t=1,\ldots,T] \)
- IRA: \( PV[\text{Annuity of Small Deposits at times } t=1,\ldots,T] = PV[\text{Large Withdrawal at retirement date } T] \)
NPV APPLICATION: RETIREMENT ACCOUNTS

Steve is 22 and anticipating retirement at 72.

How much will he have saved by then if the annual rate is \( r = 7\% \)?

- Suppose he makes 50 deposits of $1000, one at the end of each of the next 50 years.
- Let’s go about this in a roundabout way by finding the PV of this annuity first. The formula:

\[
P_{\text{ANN}} = C \left[ \frac{1}{r} - \frac{1}{r(1+r)^T} \right] = 50 \left[ \frac{1}{.07} - \frac{1}{.07(1.07)^{50}} \right] = 13332
\]

- How do we get this to age 72? Think of PV as a BLOB.
- Find the FV! Let it grow! Multiply by \((1.07)^{50}\)

\[
F_{\text{ANN}} = P_{\text{ANN}} \times (1+r)^T = C \left[ \frac{1}{r} - \frac{1}{r(1+r)^T} \right] (1+r)^T
\]

\[
= 50 \left[ \frac{1}{.07} - \frac{1}{.07(1.07)^{50}} \right] (1.07)^{50} = 13332(1.07)^{50}
\]

\[
C \left[ \frac{1}{r} - \frac{1}{r(1+r)^T} \right] (1+r)^T = C \left[ \frac{(1+r)^T - 1}{r} \right] = F_{\text{ANN}}
\]
Using the Annuity Shortcut & Blob Trick
What if you gave up coffee for 4 years and saved that money instead?

• You are 20 yrs old and plan buy a $3 coffee each day of the year (assume there are 360 days/year) for the next four years: \( T = 360 \times 4 = 1440 \).

• Suppose you could invest at a 3.6% rate (APR, compounded daily).

• Find the PV of your coffee expenditures.

\[
PV_{\text{ANNUITY}} = X, T = 1440, r = \frac{0.036}{360} = .0001 = 3 \left( \frac{1}{.0001} - \frac{1}{.0001(1.0001)^{1440}} \right) = $4023.18
\]

• You plan to retire on your 70th birthday. Suppose you open an individual retirement account and invest these dollars instead of using them to buy coffee. Find the FV of your IRA when you retire.

• TRICK: After converting the 1440 piles of money into a large blob at time \( t = 0 \), we move the entire blob 18,000 days into the future.

\[
FV = ($4023.180537)(1.0001)^{50 \times 360} = $24,336.63
\]
APPLY: A Bond has a Different Repayment Schedule

• Once students understand loans, it’s not hard to switch gears and take about bonds, which are like loans that are repaid with one large sum (at maturity) preceded by a series of regular interest payments called coupons.

• Loan: $PV[\text{Large Withdrawal at } t=0] = PV[\text{Annuity of Small Deposits at times } t=1,\ldots,T]$

• Bond: $PV[\text{Large Withdrawal at } t=0] = PV[\text{Annuity of Regular Coupon Payments at times } t=1,\ldots,T] + PV[\text{Large Repayment of Face Value at Maturity Date } T]$
Valuing A Simple Bond

Consider a par $1000, one-year, 6% semi-annual APR (twice per year) bond

1st, check the date it matures (in 1 year) and the face value ($1000).

2nd, use the coupon rate to set # periods (T=2 six-month periods).

3rd, convert the coupon rate (in APR form) to the effective periodic rate: 6% per year (APR, compounded semi-annually) → 3% every six months.

4th, compute the coupon: (periodic rate)*(par)=(.03)($1000)=$30.

5th, draw a timeline:

6th, compute the present value:

\[
PV(Bond) = PV(Coupons) + PV(Face)
\]

\[
PV = \frac{30}{1+r} + \frac{30}{(1+r)^2} + \frac{1000}{(1+r)^2}
\]
How To Value A Typical Bond

\[ P_{BOND} = PV_{COUPONS} + PV_{FACE} \]

\[ = \frac{cF}{(1 + r)} + \frac{cF}{(1 + r)^2} + \ldots + \frac{cF}{(1 + r)^T} + \frac{F}{(1 + r)^T} \]

\[ = cF \left[ \frac{1}{r} - \frac{1}{r(1 + r)^T} \right] + \frac{F}{(1 + r)^T} \]

**NOTE:** Long term bonds may involve a lot of coupons, so instead of finding the PV of each, we use an annuity formula short-cut to find the PV of all of them at once!

When I lend money by purchasing a bond, the borrower promises to repay me with a series of regular interest payments and a final large repayment of the face value.

Because the size and timing of these payments are established by the bond certificate (remember – it is printed in indelible ink!!!), it makes sense we would call this a fixed income security. Many investors like this predictability.

Each interest payment is known as a coupon and computed this way:

**Coupon=(coupon rate)*Face=cF**
The Risks of Bond Investing

- “A bird in the hand is worth two in the bush.”
- We don’t like “hazard; peril; exposure to loss or injury”
- We must assess the probability that a event occurs
- It is risky to buy bonds. We trade away a sure pile of money today for a dubious promise of fixed future payments only if we are sufficiently compensated for:
  - **INFLATION RISK**: Rising prices erode purchasing power of $1
  - **DEFAULT RISK**: What if borrower doesn’t repay the lender?
  - **INTEREST RATE RISK**: What if we lend today at 6% and then rates promptly rise to 7%, so other lenders would be charging and earning more than us? Our 6% bond would be less valuable than their 7% bonds! Bond prices and discount rates are inversely related.
  - **LIQUIDITY RISK**: In case we need cash in a hurry for some emergency, what if we can’t easily sell this bond?
  - **POLITICAL RISK**: What if unfavorable tax changes sap gains? Is there a chance some government will arbitrarily seize my asset?!
Breaking Down An Interest Rate

• An interest rate compensates a bond investor for waiting and worrying. If we use $\rho$ (Greek letter rho) to denote a risk premium, then we could think of decomposing an interest rate into a risk-free rate plus a sum of risk premiums, one to compensate an investor for each separate type of risk:

$$r_{Frederick\ Bond} = r_{Risk-Free} + \rho_{Inflation} + \rho_{Default} + \rho_{Interest\ Rate} + \rho_{Liquidity} + \rho_{Political}$$

• Some bonds are very safe. For instance, 3-month U.S. Treasury bills have no default risk (govt can print dollars!), no liquidity risk (very actively traded), and mature so quickly that there’s little time for much to happen re: inflation, rising rates or tax changes. Logically, the return on T-bills is tiny! TIP: $r_{Risk-Free} \approx r_{T-Bills}$

• To determine whether a particular bond is relatively safe, we check its bond rating … and hope the rating is accurate.
Some Insights From Bond Pricing

- The price of a bond is determined by 4 variables. The first 3 are printed in indelible ink and unchanging: face value $F$, effective coupon rate $c$, and # periods $T$. Thus, the action in the bond mkt boils down to changes in the 4th variable - the discount rate $r$. This is the RISK-ADJUSTED return that makes this bond “fairly” priced. If a bond suddenly seems riskier, its price must fall.

- As shown below, we could think about decomposing a discount rate into the waiting (impatience) part and then several worrying parts (various risk premiums)

- When bond investors are concerned about rising inflation, more frequent defaults, rising rates, difficulty selling bonds, or future tax hikes, then $r$ increases and $P_{BOND}$ falls. This is what happened in 2008!

$$P_{BOND} = PV_{COUPONS} + PV_{FACE}$$

$$= \frac{cF}{(1 + r)} + \frac{cF}{(1 + r)^2} + ... + \frac{cF}{(1 + r)^T} + \frac{F}{(1 + r)^T}$$

$$= cF \left[ \frac{1}{r} - \frac{1}{r(1 + r)^T} \right] + \frac{F}{(1 + r)^T}$$

$$r_{Bond} = r + \rho_{Risk-Free} + \rho_{Inflation} + \rho_{Default} + \rho_{Interest Rate} + \rho_{Liquidity} + \rho_{Political}$$
LEAP: Shares Feature Dividends & Resale

- Loan: \( PV[\text{Large Withdrawal at } t=0] = PV[\text{Annuity of Small Deposits at times } t=1,\ldots,T] \)
- Bond: \( PV[\text{Large Withdrawal at } t=0] = PV[\text{Annuity of Regular Coupon Payments at times } t=1,\ldots,T] + PV[\text{Large Repayment of Face Value at Maturity Date } T] \)
- Share: \( PV[\text{Large Withdrawal at } t=0] = PV[\text{Series of Nonconstant, Irregular Dividends}] + PV[\text{Large Inflow from Reselling Share at time } T] \)
Today’s Takeaway: The Loan Formula

- One formula relates the amount borrowed (B), the interest rate (r), the number of repayment periods (T), and the monthly payment (X).

\[ B = X \left[ \frac{1}{r} - \frac{1}{r(1+r)^T} \right] \Rightarrow \$200,000 = X \left[ \frac{1}{0.005} - \frac{1}{0.005(1.005)^{360}} \right] \Rightarrow X = \$1199.10 \approx \$1200 \]

- Armed with this formula we have an opportunity to do wondrous things for our students. We can help them
  - Develop financial savvy
  - Understand role of finance in macro and micro
  - Understand incentives faced by both borrowers and lenders
  - See the value in learning (or reviewing) a bit of ECON before borrowing
  - Be prepared to think about IRAs, bonds, stocks, and other projects
And another thing

• Armed with a basic mortgage formula, a **macroeconomic** instructor can more easily explain subprime mortgages, adjustable rate mortgages, Freddie Mac and Fannie Mae, mortgage-backed securities, the demise of Bear Stearns and Lehman Brothers, toxic assets, TARP, the 2008 credit crunch, tightening of lending standards, and monetary policy by the Fed in the Bernanke and Yellen years.

• Armed with a basic mortgage formula, a **microeconomic** instructor can explain that while a family borrows to buy a house, a firm may borrow to build a production facility (e.g., Tesla’s gigafactory) or buy other inputs, and both types of borrowers care about their credit records and the costs of borrowing. Moreover, borrowing $$$ for education to enhance one’s human capital and ability to repay the loan may be compelling while borrowing $$$ to buy potato chips might be unwise.

• Advanced courses might apply this present value of annuity formula to study share prices (using an annuity of expected dividends), bond prices (using an annuity of coupon payments), retirement accounts (using an annuity of regular deposits), or intellectual property rights (using an annuity of expected royalty payments).